Totally Invariant State Feedback Controller for Position Control of Synchronous Reluctance Motor

Kuo-Kai Shyu, *Member, IEEE*, Chiu-Keng Lai, and John Y. Hung, *Senior Member, IEEE*

***Abstract*—A new totally invariant state feedback controller is designed by combining the classical state feedback controller and the variable-structure control (VSC). The combination of these two different control methods has the advantages of both their merits: 1) the easy design of the state feedback and 2) the strong robustness of the VSC. In other words, the system performance can be simply designed for the nominal system by using the classical state feedback, which includes such well-known techniques as the pole placement or the linear quadratic method. Then, VSC is used to ensure the control effect. To demonstrate the effectiveness of the totally invariant state feedback controller, it is applied to the position control of a synchronous reluctance motor. Simulation results are first given. In addition, a prototype hardware system is built and experimentally evaluated.**

***Index Terms*—Position control, state feedback, synchronous reluctance motor, variable-structure control.**

I. INTRODUCTION

IN THE PAST decade, variable-structure control (VSC) or sliding-mode control (SMC) strategies have been the focus of many studies for the control of dc and ac servo drive systems [1]–[5] because they can offer many properties such as insensitivity to parameter variations, external disturbance rejection, and fast dynamic response. Generally, the typical cascade structure control for the position control of electric drives is adopted. In this control structure, the motor drive acts as a torque actuator. Therefore, the position control of electric drives can be simplified to be a mechanical system, whose control input is the torque, which can be produced by any torque actuator (dc motor, permanent-magnet ac motor, induction motor, etc.). Considering the mechanical system whose model has torque input, much research has used VSC method to design a controller for the system [1]–[5]. In this paper, a newly developed VSC called a totally invariant state feedback controller is also designed for a mechanical system, whose model is driven by a torque input.

Generally, to design a VSC system, two design phases must be considered, namely, the reaching phase and the sliding phase. The robustness of a VSC system resides in its sliding phase, but not in its reaching phase. In other words, the closed-loop system dynamics are not completely robust all the time. In addition, it is not easy to shape the dynamics of the reaching phase.

From the designers' viewpoint, linear state feedback control is theoretically an attractive method for controlling a linear plant. The method has the properties of the flexibility of shaping the dynamics of the closed-loop system to meet the desired specification. Techniques such as pole placement or linear quadratic method can be used to achieve the design objectives. For example, motor speed and position are taken as the system states. Therefore, one can easily use classical state feedback control design to shape the desired performance. Motor systems, however, are subjected to parameter variations and load disturbances, which make the system model uncertain and disturbed. In this case, it is difficult to design a classical state feedback control to satisfy the performance.

Thus, if one wants to develop an effective control strategy for uncertain motor systems with state feedback or VSC, one has to address problems of: 1) robustness; 2) keeping the designing flexibility of classical state feedback control; and 3) solving the reaching phase problem of the VSC. Consequently, the proposed method should combine the advantages of the classical state feedback control and the VSC method. Thus, the proposed totally invariant state feedback controller possesses the full flexibility of state feedback control in shaping the closed-loop dynamics, and it also has the full robustness characteristics of VSC.

Traditionally, the synchronous reluctance motor (SynRM) has been regarded as dynamically inferior to other types of rotating electric machines. Accordingly, its primary use has been limited to variable-frequency applications using open-loop control, such as fiber spinning machines and pumps. When compared with other areas, however, the SynRM exhibits advantages due to its simple, robust construction and relatively simple control electronics. For example, its mechanical simplicity is marked by the absence of slip rings, brushes, and dc field windings. From a control standpoint, the SynRM is simpler than the induction motor, which requires computation of slip in high-performance servo applications. In view of these advantages and recent progress in machine design and power electronics, much research has devoted fresh attention to the control of the SynRM [7]–[12]. To demonstrate the effectiveness of the proposed controller, the totally invariant state feedback controller is applied to the position control of the SynRM. Simulations and experiments are made to show the validity.

Manuscript received May 9, 2000; revised January 3, 2001. Abstract published on the Internet February 15, 2001. This work was supported by the National Science Council of Taiwan, R.O.C., under Contract NSC 89-2213-E008-042.

K.-K. Shyu and C.-K. Lai are with the Department of Electrical Engineering, National Central University, Chung-Li, Taiwan 320, R.O.C. (e-mail: kkshyu@sun1.ee.ncu.edu.tw).

J. Y. Hung is with the Electrical and Computer Engineering Department, Auburn University, Auburn, AL 36849-5201 USA.

Publisher Item Identifier S 0278-0046(01)03382-2.

This paper is organized as follows. First, in Section II, the concepts of totally invariant state feedback control are introduced. Second, the SynRM model and basic torque control methods are presented in Section III. Next, SynRM position control using totally invariant state feedback control is given in Section IV. The design procedure for the performance requirements is also shown in this section. Then, Section V gives several simulation studies compared with conventional VSC and the newly designed totally invariant state feedback controller. Experimental setups and results to practically prove the effectiveness of the proposed control system are given in Section VI. Finally, conclusions are given in Section VII.

II. Totally Invariant State Feedback Control [6]

To consider a single-input linear system in its nominal condition expressed in controllable canonical form,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \tag{1}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & \cdots & -a_n \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}. \tag{2}$$

In (1), $\mathbf{x} \in R^n$ is the state vector, u is the scalar control input, and $\mathbf{A}$ and $\mathbf{b}$ have appropriate dimensions. The sign of parameter b is known.

Under the perturbed condition, (1) becomes

$$\dot{\mathbf{x}} = (\mathbf{A} + \triangle\mathbf{A})\mathbf{x} + (\mathbf{b} + \triangle\mathbf{b})u + \mathbf{d} \tag{3}$$

where $\triangle\mathbf{A}$ and $\triangle\mathbf{b}$ are the perturbations in $\mathbf{A}$ and $\mathbf{b}$, respectively, and $\mathbf{d} \in R^n$ represents the external disturbance. To maintain controllability, it is assumed that $\operatorname{sgn}(\mathbf{b} + \triangle\mathbf{b}) = \operatorname{sgn}(\mathbf{b})$. Equation (3) can be expressed in the form of

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{p}. \tag{4}$$

$\mathbf{p} \in R^n$ is the total perturbation given by

$$\mathbf{p} = \triangle\mathbf{A}\mathbf{x} + \triangle\mathbf{b}u + \mathbf{d}. \tag{5}$$

The control u can be designed in two parts: u_L is a linear component, and u_{VSC} is the VSC component. First, let the nominal system be under linear state feedback control, that is,

$$u_L = -\mathbf{k}^{\mathrm{T}}\mathbf{x}, \quad \mathbf{k}^{\mathrm{T}} = [k_1\ k_2\ \cdots\ k_n] \tag{6}$$

where $\mathbf{k}$ is the feedback gain vector, which can be obtained using linear control design. Some techniques in this category include pole placement and linear quadratic design. The closed-loop dynamic, in the nominal condition, is given by

$$\dot{\mathbf{x}} = [\mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}}]\mathbf{x} = \mathbf{A}_c \mathrm{x} \tag{7}$$

with

$$\mathbf{A}_c = \mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\alpha_1 & -\alpha_2 & \cdots & -\alpha_n \end{bmatrix} \tag{8}$$

where

$$\alpha_i = a_i + bk_i, \quad i = 1 \ldots n.$$

Next, consider a scalar function

$$\sigma(\mathbf{x}, t) = \mathbf{c}^{\mathrm{T}}[\mathbf{x} - \mathbf{x_0}] - \mathbf{c}^{\mathrm{T}}\mathbf{A}_c \int_0^t \mathbf{x}(\tau)d\tau. \tag{9}$$

Based on (7), it is obvious that $\sigma(\mathbf{x}, t) = 0$ under the nominal condition. Therefore, for any initial condition $\mathbf{x}_0$ and any state feedback (6), the system (1) possesses a sliding surface $\sigma(\mathbf{x}, t) = 0$ on which the state slides. It can be easily proved that the perturbed system (4) under the condition $\sigma(\mathbf{x}, t) = 0$ reserves an equivalent system dynamic the same as the closed-loop dynamic in the nominal condition given in (7) [6].

When perturbation $\mathbf{p}$ is present, the control u_L will not be able to maintain the sliding mode. Additional control effort is needed to keep the states on the sliding surface. To control the states on the sliding surface under the perturbed condition, this extra control effort is given as $u_{VSC} = -q\operatorname{sgn}(\sigma)$. Then, the resultant control is

$$u = u_L + u_{VSC} = -\mathbf{k}^{\mathrm{T}}\mathbf{x} - q\operatorname{sgn}(\sigma). \tag{10}$$

The added term $-q\operatorname{sgn}(\sigma)$ is the VSC for the system and σ is its switching function. The VSC is used to meet the curbing condition, $\sigma\dot{\sigma} < 0$ if $\sigma \neq 0$, so that the closed-loop system dynamics for the nominal condition can be obtained even though the perturbations exist.

III. SynRM Modeling and Torque Control

The d–q-axes equations for the SynRM are generally described by [13]

$$v_{ds} = L_{ds}\frac{d}{dt}i_{ds} + R_s i_{ds} - \omega_e L_{qs} i_{qs} \tag{11}$$

$$v_{qs} = L_{qs}\frac{d}{dt}i_{qs} + R_s i_{qs} + \omega_e L_{ds} i_{ds} \tag{12}$$

where v_{ds} and v_{qs} are the d, q-axes stator voltages, i_{ds} and i_{qs} are the d, q-axes stator currents, L_{ds} and L_{qs} are the d, q-axes inductances, R_s is the stator resistance, and ω_e is the electric frequency.

The corresponding electromagnetic torque production is

$$T_e = \frac{3}{2}\frac{p}{2}(L_{ds} - L_{qs})i_{ds}i_{qs} \tag{13}$$

or

$$T_e = \frac{3}{4}\frac{p}{2}(L_{ds} - L_{qs}){i_s}^2\sin(2\delta) \tag{14}$$

where p is the pole number of the motor, δ is the current angle, and $i_s = \sqrt{{i_{ds}}^2 + {i_{qs}}^2}$, and

$$\begin{cases} i_{ds} = i_s\cos(\delta) \\ i_{qs} = i_s\sin(\delta). \end{cases} \tag{15}$$

The associated mechanical equations are as follows:

$$J_m\frac{d\omega_m}{dt} + B_m\omega_m = T_e - T_L \tag{16}$$

$$\frac{d\theta_m}{dt} = \omega_m \tag{17}$$

where ω_m is the rotor velocity, θ_m is the rotor angular displacement, J_m is the moment of inertia, B_m is the damping coefficient, and T_L is the load torque.

There are four torque control strategies for the SynRM [13]. Three among them use constant current angle controls. They are maximum torque control (MTC), maximum power-factor control (MPFC), and maximum rate of change of torque control (MRCTC). The fourth one, constant current in inductive axis control (CCIAC), uses the constant direct current control. A brief review of these four torque control strategies is given below.

First, consider the MTC strategy, which sets the current angle δ to $\delta = 45°$. Since $\sin(2\delta) = \sin(90°) = 1$, (14) becomes

$$T_e = \frac{3}{4}\frac{p}{2}(L_{ds} - L_{qs}){i_s}^2 \tag{18}$$

or

$$T_e = K_1{i_s}^2 \tag{19}$$

where $K_1 = (3/4)(p/2)(L_{ds} - L_{qs})$. Note that $i_{ds} = i_{qs}$ for MTC mode, so

$$i_{ds} = \frac{\sqrt{2}}{2}i_s.$$

Observing the produced torque (19), the produced motor torque is always positive. For VSC, two opposite control torques are necessary. Therefore, (19) must be redefined to permit the production of negative torque. If the controller output is positive, it implies that the q-axis current vector must be placed ahead of the d axis 45° in accordance with the rotation direction and we take it as a positive angle. When a negative torque is required, the current angle is placed behind the d axis 45°, i.e., the current angle is set to $\delta = -45°$. In this mode, (19) can be modified as

$$T_e = -K_1{i_s}^2. \tag{20}$$

The corresponding concepts are shown in Fig. 1. It can be seen that if the MTC torque control strategy is used, one has only to control the angle and magnitude of the current vector to match the desired torque.

As for the other two constant current angle modes, (MPFC and MRCTC), when they are applied to VSC-based motor control, one only needs to adjust the current angle δ. That is attributed to the fact that the concepts of these two control strategies are the same as those of MTC mode except with a different current angle. For example, if MPFC mode is used, then [13]

$$\delta = \tan^{-1}\left(\sqrt{\frac{L_{ds}}{L_{qs}}}\right) \tag{21}$$

and if MRCTC mode is adopted, then

$$\delta = \tan^{-1}\left(\frac{L_{ds}}{L_{qs}}\right) \tag{22}$$

must be chosen.

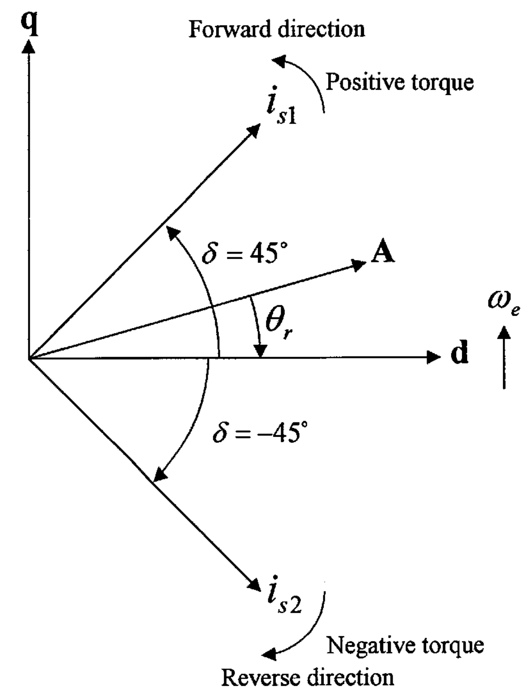

Fig. 1. Positive and negative torque current vectors of maximum torque control of synchronous reluctance motor.

When the constant direct current control, CCIAC mode, is selected, then (13) can be written as

$$T_e = K_2 i_{qs} \tag{23}$$

where $K_2 = (3/2)(p/2)(L_{ds} - L_{qs})i_{ds}$. This is what one calls vector control with i_{qs} proportional to torque. If the d-axis current is kept as a constant, then one only needs to control i_{qs} to get the desired torque. Thus, if the VSC controller outputs a torque command T^*, which is proportional to i_{qs}, then the current angle can be decided by

$$\delta = \tan^{-1}\left(\frac{T^*}{i_{ds}}\right). \tag{24}$$

As mentioned above, four control modes can be used to produce the desired torque by setting the magnitude and angle of the stator current. Here, MTC is adopted for our SynRM torque control mode, because this torque control mode can be easily implemented and has the property of maximum torque per ampere generation.

IV. SynRM Position Control by Totally Invariant State Feedback Controller

For position control using state feedback, one has to describe the system model in state variable form. The corresponding SynRM dynamic equations in the state-space model are expressed in (25) as

$$\begin{bmatrix}\dot{\theta}_m \\ \dot{\omega}_m\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -\frac{B_m}{J_m}\end{bmatrix}\begin{bmatrix}\theta_m \\ \omega_m\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{J_m}\end{bmatrix}T_e - \begin{bmatrix}0 \\ \frac{1}{J_m}\end{bmatrix}T_L \tag{25}$$

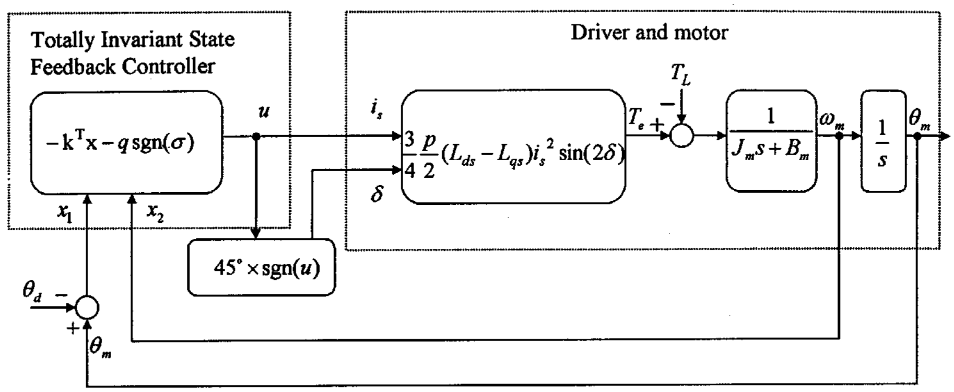

Fig. 2. Totally invariant state feedback control and maximum torque control strategy-based synchronous reluctance motor position control system.

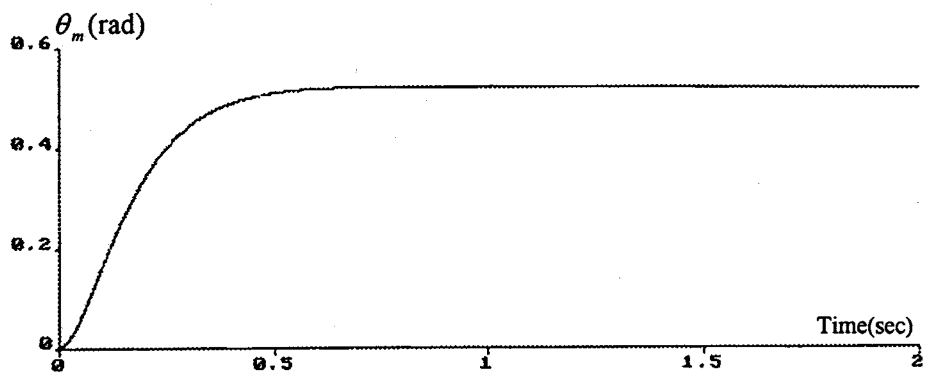

Fig. 3. Position response of the nominal closed-loop SynRM position control system.

and the electromagnetic torque equation based on MTC torque control mode is given as

$$T_e = K_1 i_s{}^2 \sin(2\delta) \tag{26}$$

where K_1 has been defined in (19), $\delta = +45^\circ$ for a positive torque, and $\delta = -45^\circ$ for a negative torque. Note that the totally invariant state feedback controller is simply designed for a mechanical system whose model is given by (25), where the control input is the torque which can be produced by any torque actuator.

For a desired rotor position θ_d, one first defines the position error and its derivative as

$$\begin{cases} x_1 = \theta_m - \theta_d \\ x_2 = \omega_m. \end{cases} \tag{27}$$

Inserting (26) and (27) into (25), we have the error dynamical equations as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} i_s{}^2 \sin(2\delta) - \begin{bmatrix} 0 \\ T_L' \end{bmatrix} \tag{28}$$

where $a = B_m/J_m$, $b = K_1/J_m$, and $T_L' = T_L/J_m$. Compared with (2), the corresponding matrix $\mathbf{A}$ and vector $\mathbf{b}$ are, respectively,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ b \end{bmatrix}. \tag{29}$$

Since the state feedback control is adopted, based on the MTC torque control, the linear control component is

$$u_L = -\mathbf{k}^{\mathrm{T}}\mathbf{x} = i_s{}^2 \sin(2\delta). \tag{30}$$

Then, torque (26) can be rewritten as $T_e = -K_1[\mathbf{k}^{\mathrm{T}}\mathbf{x}]$.

First, the pole placement of the linear state feedback is used to design the system characteristics for SynRM position control without thinking of the uncertainties and disturbances. Thus, the feedback gain $\mathbf{k}$, $\mathbf{k}^{\mathrm{T}} = [k_1\ k_2]$, is chosen for the nominal system such that it will exhibit as a second-order system with characteristic equation

$$s^2 + (a + bk_2)s + bk_1 = 0 \tag{31}$$

where a and b are defined in (28). In this condition, the system's poles are located at

$$s_{1,2} = \frac{-(a + bk_2) \pm \sqrt{(a + bk_2)^2 - 4bk_1}}{2}. \tag{32}$$

A proper choice for k_1 and k_2 will determine the system responses to satisfy such requirements as damping, rise time, etc.

Second, the switching surface $\sigma(\mathbf{x}, t)$ in (9) can be decided in the following. Matrix $\mathbf{A}_c$ is the equivalent system matrix under the state feedback control for the nominal system. Vector $\mathbf{c}^{\mathrm{T}}$ can be simply decided through the choice of $\mathbf{c}^{\mathrm{T}}\mathbf{b} = 1$. Because the

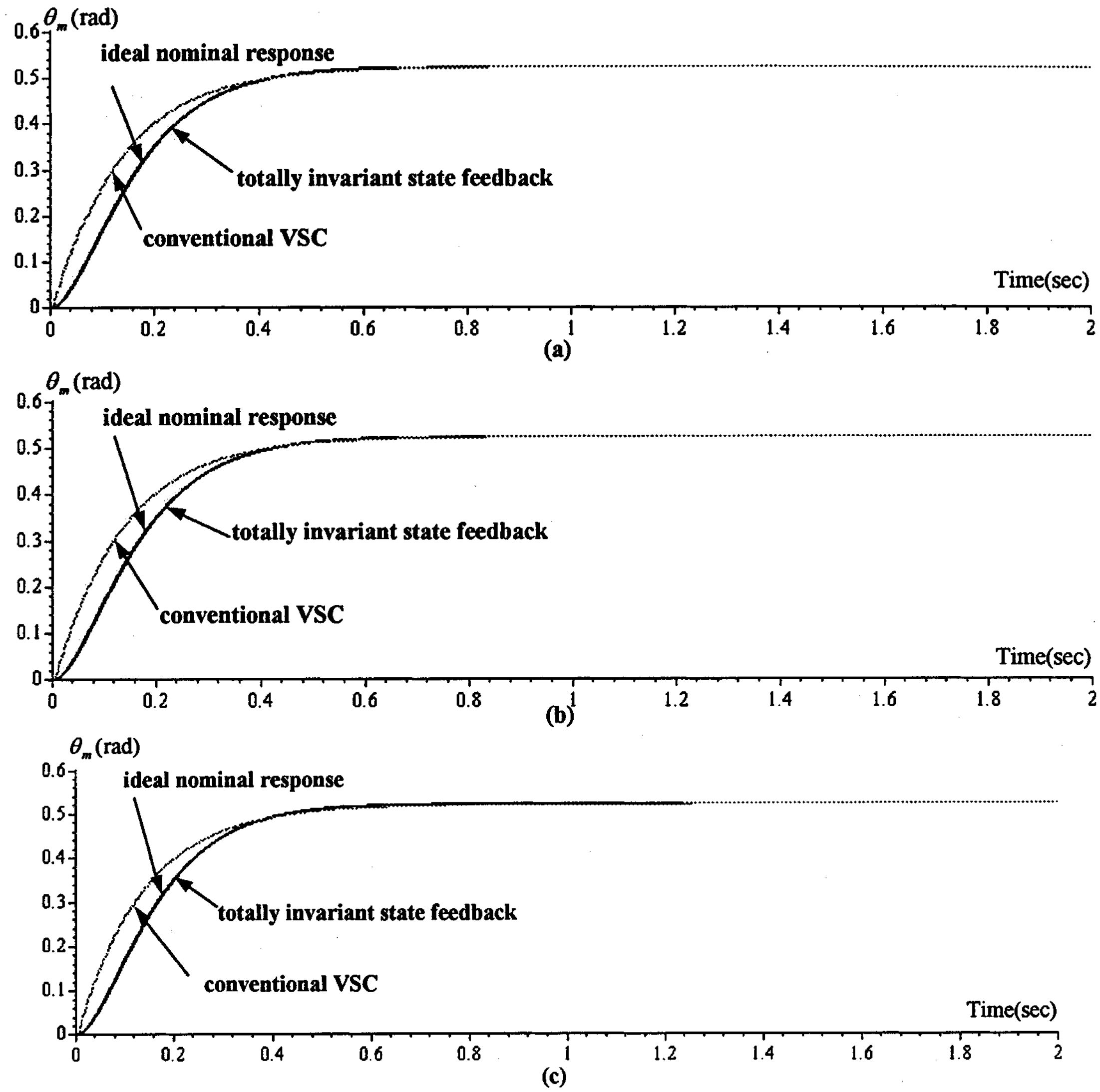

Fig. 4. Simulated position responses of SynRM system controlled by the conventional VSC and the proposed totally invariant state feedback control, respectively. (a) Nominal system. (b) System with L_{ds} 20% variation. (c) System with sudden load added at 0.1 s and removed at 1.2 s.

matrix $\mathbf{b}$ of the SynRM system is $\mathbf{b}^{\mathrm{T}} = [0 \quad b]$, then, $\mathbf{c}$ can be set as

$$\mathbf{c}^{\mathrm{T}} = \begin{bmatrix} 0 & \frac{1}{b} \end{bmatrix}. \tag{33}$$

The VSC component $-q\,\mathrm{sgn}(\sigma)$ is used to overcome the lumped uncertain parameters and external disturbance. For the motor position control system, the total perturbation $\mathbf{p}$ is a form such as $\mathbf{p}^{\mathrm{T}} = [0 \quad p]$. Therefore, the magnitude of q should satisfy

$$q > |p|_{\max}. \tag{34}$$

Thus, the reaching condition, $\sigma\dot{\sigma} < 0$ if $\sigma \neq 0$, can be always ensured. The block diagram of SynRM position control using totally invariant state feedback control is shown in Fig. 2.

One should note that the above derivations do not consider the limitations of the torque and speed. Therefore, when considering the practical case, the following remark should be considered.

Remark 1: It should be noted that the proposed linear behavior of the controlled motor is significative for small movements (i.e., when the torque and speed saturation can be neglected).

V. Simulation Results

In order to verify the proposed control strategy, simulations are first done using SIMNON software. The characteristics of the SynRM used in the simulations are given in the Appendix. The control objective is to drive the motor rotor to rotate 0.5235 rad, which is about 30°. Classical state feedback control is used to design the desired performance. Meanwhile, a conventional VSC-based controller as well as the proposed totally invariant state feedback controller are applied to the SynRM position control for comparison. For the position control of the SynRM, a critical damped response will be considered. The dynamic equa-

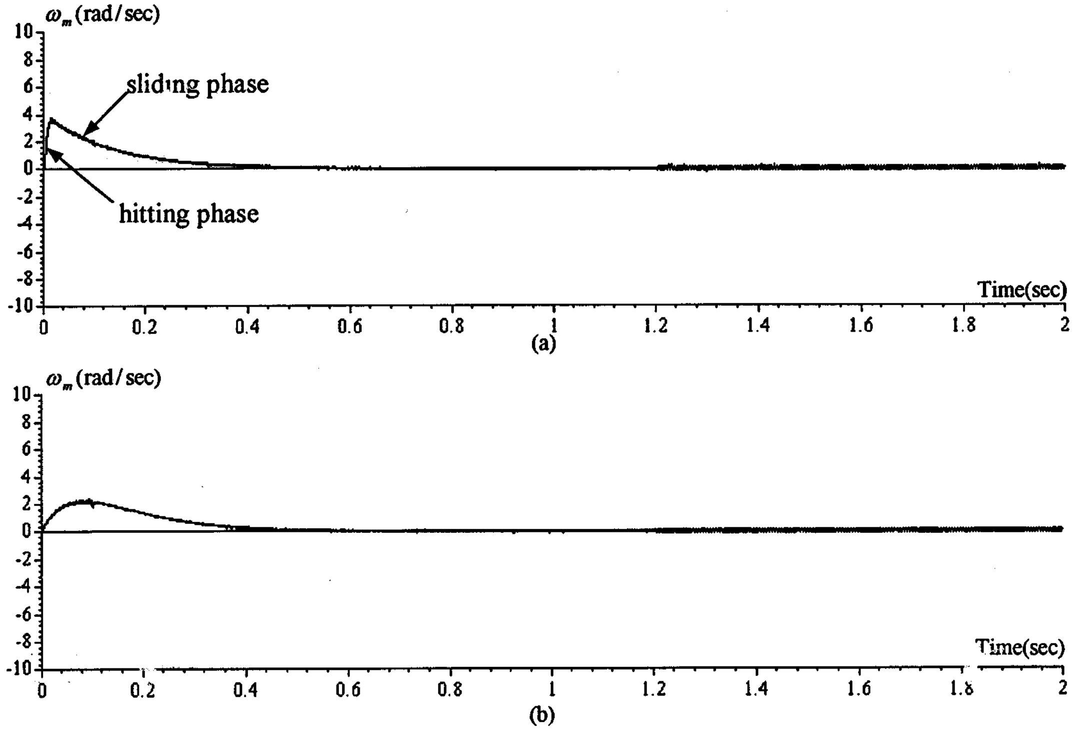

Fig. 5. Simulated speed responses of system with sudden load added at 0.1 s and removed at 1.2 s. (a) Conventional VSC. (b) Totally invariant state feedback control.

tion of the SynRM drive system given in (28) with parameters shown in the Appendix is given as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -0.2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 12.75\end{bmatrix}u-\begin{bmatrix}0\\ 100\end{bmatrix}T_L \tag{35}$$

where $u = {i_s}^2 \sin(2\delta)$.

The system given in (35) under the state feedback control with $T_L = 0$ is

$$\dot{\mathbf{x}}=\begin{bmatrix}0 & 1\\ -12.75k_1 & -0.2-12.75k_2\end{bmatrix}\mathbf{x}=\mathbf{A}_c\mathbf{x}. \tag{36}$$

The corresponding characteristic equation is

$$s^2+(0.2+12.75k_2)s+12.75k_1=0. \tag{37}$$

To have a critical damped response, a feedback gain vector $\mathbf{k}^{\mathrm{T}} = [k_1 \quad k_2] = [10.0 \quad 1.76]$ is chosen to achieve this requirement. The nominal system which is controlled by

$$u=-\mathbf{k}^{\mathrm{T}}\mathbf{x}=-[10.0 \quad 1.76]\mathbf{x}$$

will exhibit a response with rise time 0.29 s. Based on the classical state feedback control, the proposed totally invariant state feedback controller is also used to compare the control effects. An auxiliary sliding-mode surface $\sigma(\mathbf{x},t) = \mathbf{c}^{\mathrm{T}}[\mathbf{x}-\mathbf{x}_0]+\mathbf{A}_c\int\mathbf{x}d\tau$ and an extra force $-q\,\mathrm{sgn}(\sigma)$ are added to the state feedback controller to achieve the proposed robust control scheme. A corresponding controller which is designed by a conventional VSC controller is also used to evaluate and compare with the performance of the proposed controller. The conventional VSC controller is designed with a switching surface

$$\sigma(\mathbf{x})=7.535x_1+x_2=0 \tag{38}$$

which has a similar rise time as classical state feedback control. Simulation results are presented to evaluate the control performance of these two control schemes. First, the position response of a nominal closed-loop system is shown in Fig. 3, which shows the ideal response of the controlled system. In the following, the control effects of two different VSC controllers, controlling the nominal system, uncertain system, and loaded system, will be used to demonstrate the control responses.

At first, the simulated position responses of the nominal system controlled by two different VSC controllers are given in Fig. 4(a). To evaluate robustness against uncertainty, the position responses of the uncertain system controlled by the same controllers are shown in Fig. 4(b). Here, a 20% variation of L_{ds} is assumed, Finally, to evaluate robustness against an external load, a sudden load with 1.0 N·m is applied at the time 0.1 s and is removed at the time 1.2 s. The corresponding position responses are shown in Fig. 4(c). From Fig. 4(a)–(c), it can be seen that the two VSC-based control systems are robust to uncertainty and external load.

From the simulation results, it is clear that the conventional VSC approach and the proposed totally invariant state feedback control approach share an excellent robustness property. Also, the totally invariant state feedback control can be used for any reference trajectory with known and finite time derivative by using the "error model" approach, which is designed for the conventional VSC [14]. The main difference between the conventional VSC and the totally invariant state feedback control is

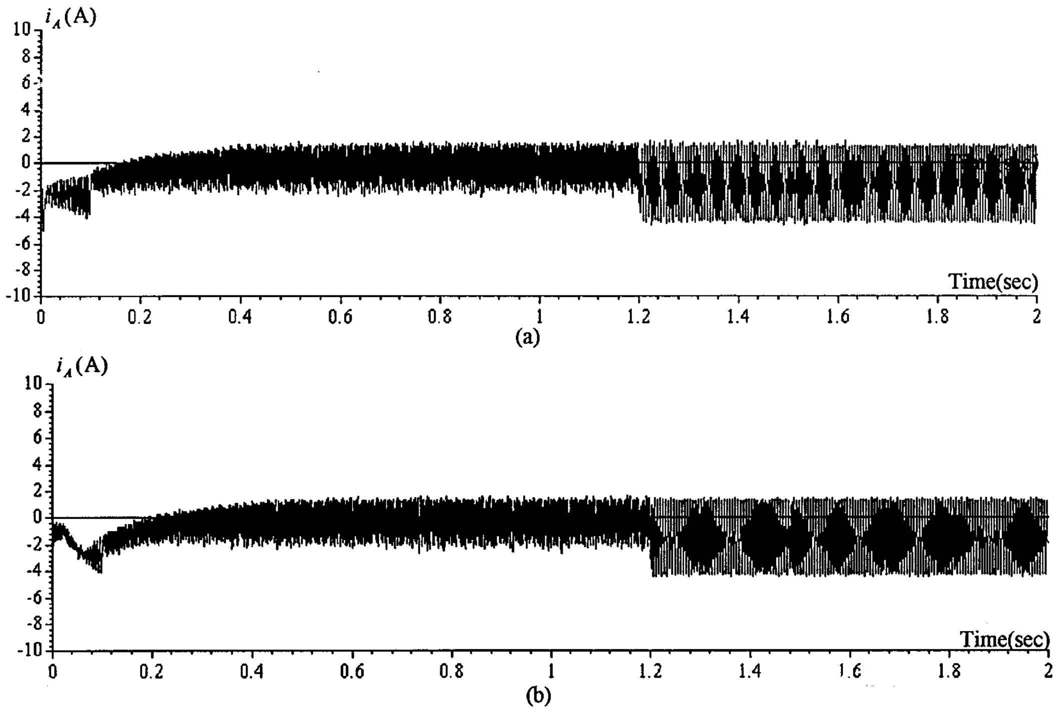

Fig. 6. Simulated phase current responses of system with sudden load added at 0.1 s and removed at 1.2 s. (a) Conventional VSC. (b) Totally invariant state feedback control.

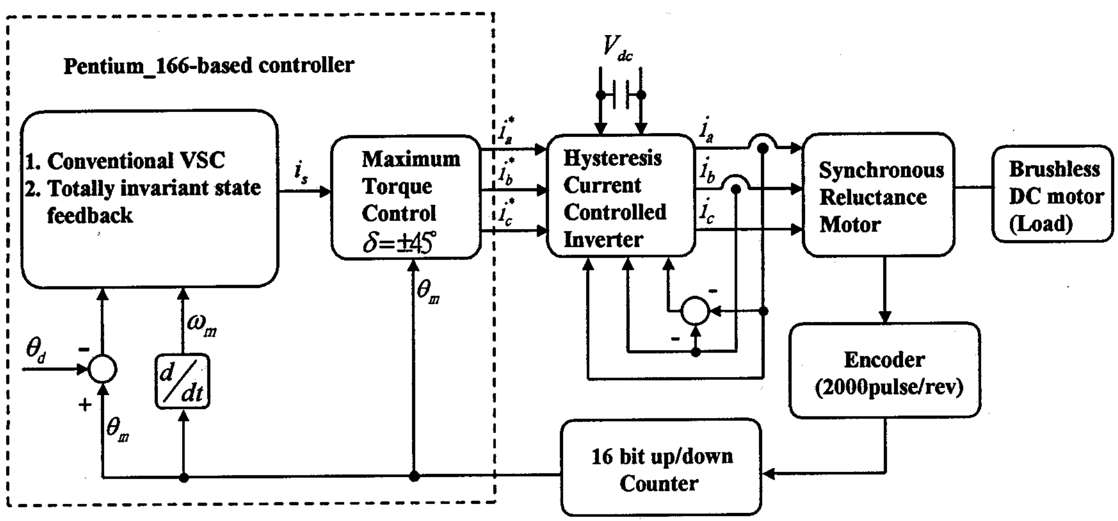

Fig. 7. Pentium-166-based synchronous reluctance motor position control system.

that the second one is always in the sliding condition, while the first one can have an initial transient (vanishing in finite time) where the system is not in the sliding condition. That implies the following remark.

Remark 2: With the totally invariant state feedback control, the error dynamics are always equal to the nominal error dynamics which have been designed shaping the sliding surface; with classical VSC, in general, this is not true since an initial transient to reach the sliding surface can be present.

Figs. 5 and 6 show the speed and current responses of the system under the same load condition as Fig. 4(c). Fig. 6(a) and (b) shows the current responses of the conventional VSC and the totally invariant state feedback control, respectively. The conventional VSC and totally invariant state feedback control include an extra force of variable-structure form, $-q\,\mathrm{sgn}(\sigma)$, which is not presented in the classical state feedback control. This variable-structure term is used to maintain the system trajectory on the switching surface or sliding mode, which makes the system robust. Thus, no steady-state error exists for the system controlled by the conventional VSC and the totally invariant state feedback control.

VI. Experimental System and Results

A. Experimental System

To practically evaluate the actual performance of the proposed control scheme, a prototype PC-based SynRM position control system is built and tested. The realized system is

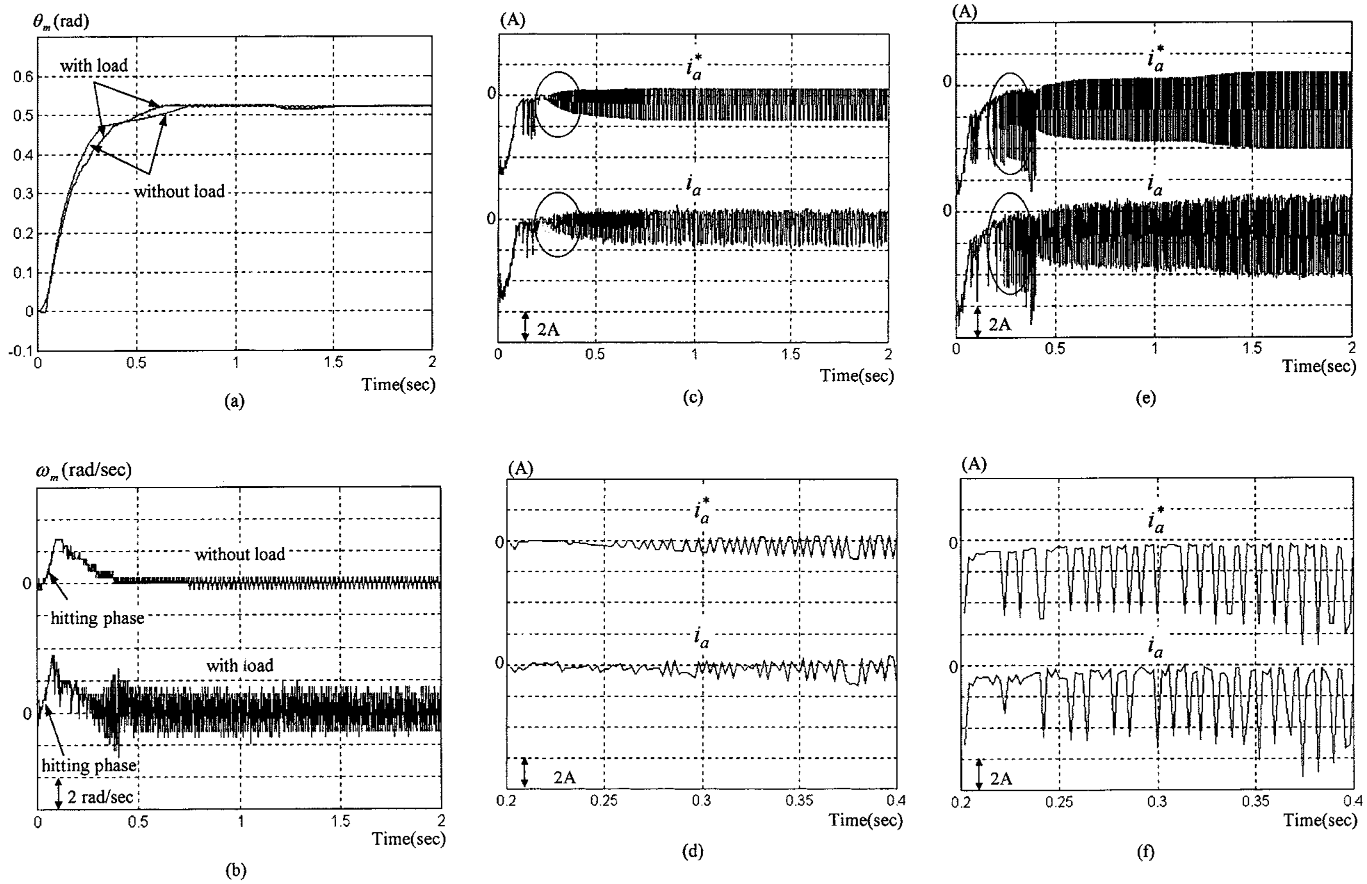

Fig. 8. Conventional VSC controlled results for the system with and without load (sudden load is added at 0.1 s and is removed at 1.2 s). (a) Position responses. (b) Speed responses. (c) Current command and controlled current response without load. (d) Zoom picture of (c). (e) Current command and controlled current response with load. (f) Zoom picture of (e).

composed of a Pentium PC, a 12-bit D/A converter, a 1.5-hp synchronous reluctance motor, and an operational-amplifier (OPA)-based hysteresis current-controlled inverter with maximum 1.8-kW output. The position control algorithms are implemented by a Pentium-166 PC with a sampling time of 0.2 ms. The position signals are sensed by a 2000-pulses/rev encoder and fed back to the PC through a 16-bit up/down counter. The corresponding mechanical velocity is computed in the PC. In order to test the feature of the proposed control scheme, a controlled external load disturbance is necessary. Thus, the experimental synchronous reluctance motor is connected to a brushless dc motor such that a controlled sudden counter torque can be directly applied to SynRM as a style of "step" load. The main program for managing data input and output is written by assembly language, and the position control strategies of the proposed totally invariant state feedback control as well as the conventional VSC are developed in the mathematical coprocessor language. The experimental data are collected by PC, and processed and printed using MATLAB software. A block diagram of this experimental system is shown in Fig. 7.

B. Results

To show the validity and effectiveness of the proposed control method, the same position control trajectories as in the simulation are adopted, i.e., a 0.5235-rad rotor displacement and a rise time of 0.29 s. The control parameters of the two VSC-based controllers are all similar to those presented in the simulations.

Figs. 8 and 9 show the corresponding position, speed, and current responses of the system with and without load for two controllers, conventional VSC, and the proposed totally invariant state feedback control. Due to the existence of uncertainty, friction, actuator deadband, external load, and so on, it can be expected that if the system is controlled by the classical state feedback control, the desired nominal response cannot be obtained. Nevertheless, as shown in Figs. 8(a) and 9(a), the position responses under the control of two VSC-based systems are robust. Owing to the existence of the reaching phase for a conventional VSC-based system, it shows a poor closeness in the acceleration period, as shown in Fig. 8(a). However, because no reaching phase exists in the totally invariant state feedback controller, position responses given in Fig. 9(a) produced by the proposed method for the cases with and without load show better closeness during the acceleration period than the conventional VSC method.

VII. Conclusions

In this paper, a new robust state feedback controller for the motor position control problem has been demonstrated. Through simulation and experimental results, it has been shown

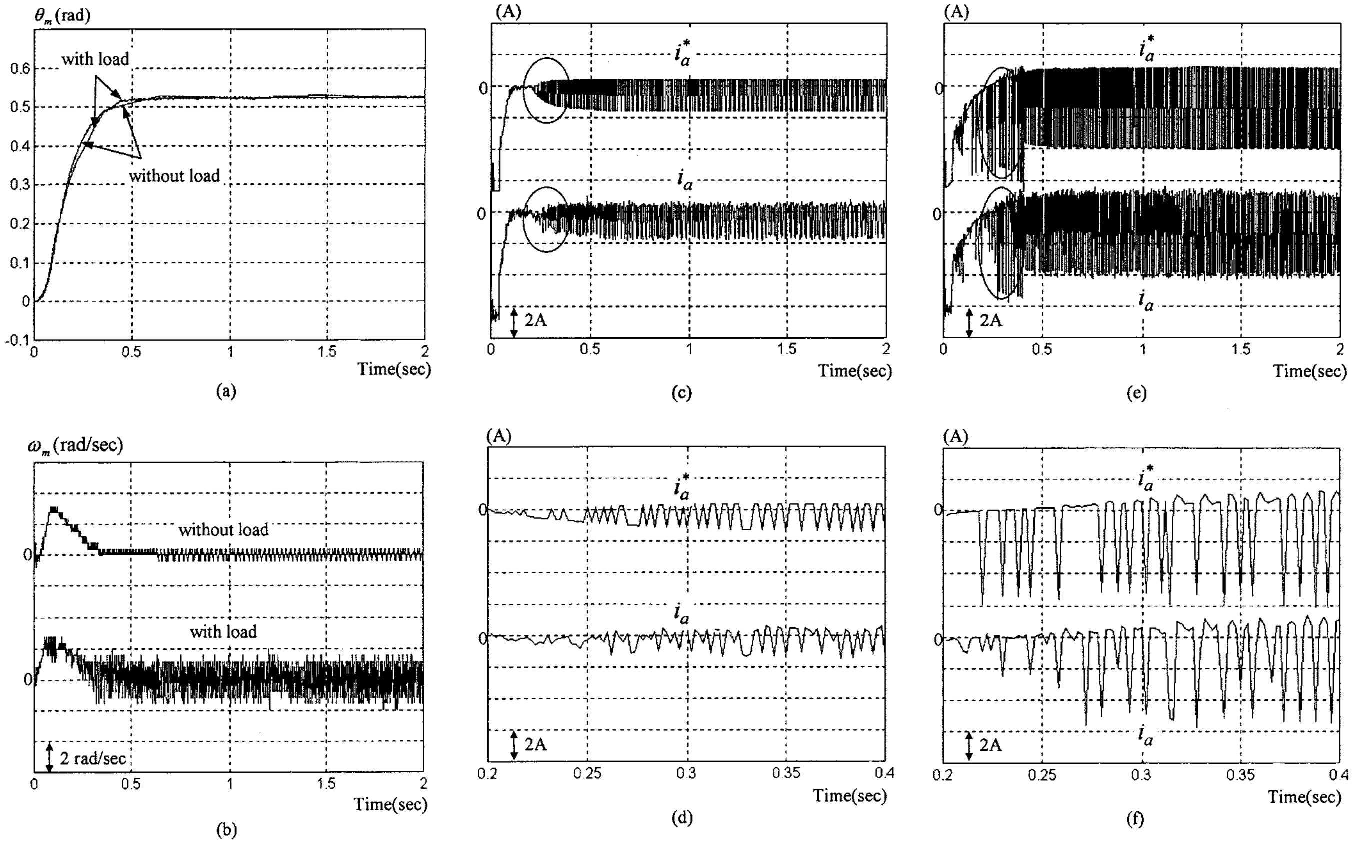

Fig. 9. Totally invariant state feedback controlled results for the system with and without load (sudden load is added at 0.1 s and is removed at 1.2 s). (a) Position responses. (b) Speed responses. (c) Current command and controlled current response without load. (d) Zoom picture of (c). (e) Current command and controlled current response with load. (f) Zoom picture of (e).

that the controlled performance under the control strategy designed by associating the classical state feedback control and the VSC control is so robust as to retain the performance as those designed according to the nominal condition. The ease of performance designed by state feedback can also be retained in practical system design by this new totally invariant state feedback controller.

Appendix
Motor Data

Rated power	1120 W;
rated voltage	230 V;
rated current	6.6 A;
direct inductanceL_{ds}	135 mH;
quadrature inductance L_{qs}	50 mH;
stator resistance R_s	0.91 Ω;
inertia J_m	0.01 N·m/s^2;
viscous coefficient B_m	0.002 N·m/s;
rated speed	1800 r/min.

References

[1] H. Hashimoto, H. Yamamoto, S. Yanagisawa, and F. Harashima, "Brushless servo motor control using variable structure control approach," *IEEE Trans. Ind. Applicat.*, vol. 24, pp. 160–170, Jan./Feb. 1997.

[2] K. W. Lim, T. S. Low, M. F. Rahman, and L. B. Wee, "A discrete time variable structure controller for a brushless DC motor drive," *IEEE Trans. Ind. Electron.*, vol. 38, pp. 102–107, Apr. 1991.

[3] T. H. Liu and M. T. Lin, "A fuzzy sliding-mode controller design for a synchronous reluctance motor drive," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, pp. 1065–1075, July 1996.

[4] K. K. Shyu and H. J. Shieh, "A new switching surface sliding-mode speed control for induction motor drive systems," *IEEE Trans. Power Electron.*, vol. 11, pp. 660–667, July 1996.

[5] ——, "Variable structure current control for induction motor drives by space voltage vector PWM," *IEEE Trans. Ind. Electron.*, vol. 42, pp. 572–578, Dec. 1995.

[6] K. K. Shyu and J. C. Hung, "Totally invariant variable structure control systems," in *Proc. IEEE IECON'97*, New Orleans, LA, 1997, pp. 1119–1123.

[7] L. Xu and J. Yao, "A compensated vector control scheme of a synchronous reluctance motor including saturation and iron losses," *IEEE Trans. Ind. Applicat.*, vol. 28, pp. 1330–1338, Nov./Dec. 1992.

[8] K. Uezato, T. Senjyu, and Y. Tomori, "Modeling and vector control of synchronous reluctance motors including stator iron loss," *IEEE Trans. Ind. Applicat.*, vol. 30, pp. 971–976, July/Aug. 1994.

[9] L. Xu, X. Xu, T. A. Lipo, and D. W. Novotny, "Vector control of a synchronous reluctance motor including saturation and iron loss," *IEEE Trans. Ind. Applicat.*, vol. 27, pp. 977–985, Sept./Oct. 1991.

[10] R. Lagerquist, I. Boldea, and T. J. E. Miller, "Sensorless control of the synchronous reluctance motor," *IEEE Trans. Ind. Applicat.*, vol. 30, pp. 673–681, May/June 1994.

[11] J. E. Fletcher, B. W. Williams, and T. C. Green, "Efficiency aspects of vector control applied to synchronous reluctance motors," in *Conf. Rec. IEEE-IAS Annu. Meeting*, 1995, pp. 294–300.

[12] T. Matsuo, A. E. Antably, and T. A. Lipo, "A New control strategy for optimum-Efficiency operation of a synchronous reluctance motor," *IEEE Trans. Ind. Applicat.*, vol. 33, pp. 1146–1153, Sept./Oct. 1997.

[13] R. E. Betz, R. Lagerquist, M. Jovanovic, T. J. E. Miller, and R. H. Middleton, "Control of synchronous reluctance machines," *IEEE Trans. Ind. Applicat.*, vol. 29, pp. 1110–1122, Nov./Dec. 1993.

[14] J. J. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

Kuo-Kai Shyu (M'98) received the B.S. degree from Tatung Institute of Technology, Taipei, Taiwan, R.O.C., and the M.S. and Ph.D. degrees from National Cheng-Kung University, Tainan, Taiwan, R.O.C., in 1979, 1984, and 1987, respectively, all in electrical engineering.

In 1988, he joined National Central University, Chung-Li, Taiwan, R.O.C., where he is currently a Professor in the Department of Electrical Engineering. From 1998 to 1999, he was a Visiting Scholar with the Electrical and Computer Engineering Department, Auburn University, Auburn, AL. His teaching and research interests include variable-structure control systems and signal processing with applications in motor control, robotics, and power electronics.

Chiu-Keng Lai received the B.S. degree in electronic engineering in 1989 from National Taiwan Institute of Technology, Taipei, Taiwan, R.O.C., and the M.S. degree in information and electronic engineering in 1991 from National Central University, Chung-Li, Taiwan, R.O.C., where he is currently working toward the Ph.D. degree in electrical engineering.

He is also currently a Lecturer in the Department of Electrical Engineering, National Chin-Yi Institute of Technology, Taichung, Taiwan, R.O.C. His interests include motor control, PC-based control systems, and digital signal processing.

John Y. Hung (S'78–M'80–SM'93) received the B.S. degree from the University of Tennessee, Knoxville, the M.S.E. degree from Princeton University, Princeton, NJ, and the Ph.D. degree from the University of Illinois, Urbana-Champaign, in 1979, 1981, and 1989, respectively, all in electrical engineering.

From 1981 to 1985, he was with Johnson Controls, Milwaukee, WI, working on microprocessor-based controllers for commercial heating, ventilation, and air conditioning systems. In 1989, he joined Auburn University, Auburn, AL, where he is currently an Associate Professor of Electrical and Computer Engineering. His teaching and research interests include nonlinear control systems and signal processing with applications in process control, robotics, magnetic bearings, electric motors, and power electronics.

Dr. Hung coauthored the 1996 Best Paper in the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS. He was an Associate Editor of the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY (1997–1998) and is an Associate Editor of the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS. He was Technical Program Co-Chair for the 2000 IEEE International Symposium on Industrial Electronics (ISIE 2000).